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Abstract
The vector supersymmetry of the two-dimensional (2D) topological BF model is extended to 2D Yang–Mills. The consequences of the corresponding Ward identity on the ultraviolet behaviour of the theory are analysed.

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1. Introduction

The relation between the two-dimensional Yang–Mills theory (2DYM) and the topological models has been an object of intensive investigations over the past years [1–4]. In spite of the lacking of local degrees of freedom, 2DYM is non-trivial when analysed from the point of view of the topological field theories, as underlined for instance by [4] within the BRST framework.

An interesting feature of the topological theories, of both Witten and Schwartz type, is the existence, besides their BRST symmetry, of an additional invariance whose generators carry a vector index. This further symmetry, called vector supersymmetry [5–7], gives rise, together with the BRST generator, to an algebra of the Wess–Zumino type which, closing on-shell on the spacetime translations, allows for a supersymmetric interpretation of the topological models [5–7]. In particular, it has been shown [8] that the existence of the vector supersymmetry is deeply related to the fact that the energy–momentum tensor can be expressed in the form of a pure BRST variation, a key feature which can be taken as the proper definition of the topological theories.

It therefore seems natural to ask ourselves whether this supersymmetric structure can also be found in the 2DYM; this being the goal of this paper. As one can easily guess, we will be able to show that this question can actually be answered in the affirmative. As a by-product, a simple understanding of the ultraviolet finiteness of 2DYM will be provided.

2. Symmetries of 2DYM

Let us start by considering the gauge-invariant Yang–Mills action in 2D Euclidean spacetime:

$$S_{YM} = -\frac{1}{4}\int \mathrm{d}^2x\; F^{a\mu\nu}F^a_{\mu\nu} \tag{2.1}$$

where $F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the field strength and g denotes the gauge coupling constant. Note that in 2D the gauge connection is dimensionless, so that the coupling constant g has dimension one.

In order to analyse the symmetries of this model, it is convenient to switch to the first-order formalism [4] by rewriting the expression (2.1) in the following form:

$$S_{YM} = S_{top} + S_\phi \tag{2.2}$$

where

$$S_{top} = \tfrac{1}{2}\int \mathrm{d}^2x\; \varepsilon^{\mu\nu}F^a_{\mu\nu}\phi^a \tag{2.3}$$

and

$$S_\phi = \tfrac{1}{2}\int \mathrm{d}^2x\; \phi^a\phi^a \tag{2.4}$$

with ϕ^a being an auxiliary scalar field. Expression (2.2) is obviously seen to be equivalent to (2.1) upon elimination of the auxiliary field ϕ^a through the equations of motion. It worth remarking here that the use of the first-order formalism allows us to interpret to some extent the 2DYM as a deformation of a topological field theory, as it is easily recognized that the term S_{top} in equation (2.3) is the action of the two-dimensional topological BF model [9]. The second term S_ϕ is metric-dependent, and therefore plays the role of the deformation. Expression (2.2) also suggests what our strategy will be in order to establish the vector susy Ward identity for the 2DYM. To this end we recall that the first term S_{top} of equation (2.2), identifying a topological field theory, possesses the vector supersymmetry [9] which, however, will not leave the second term invariant. Nevertheless, it is a remarkable property of the 2DYM that the breaking terms stemming from the non-invariance of the metric-dependent part S_ϕ of the action (2.2) can be taken into account by the introduction of a suitable set of external fields. As we shall see, this procedure will enable us to write down an off-shell Ward identity which is an extension of the vector susy Ward identity of the topological two-dimensional BF model [9]. This identity will strongly constrain the ultraviolet behaviour of the 2DYM.

Once the classical counterpart of the theory is established, our next step is to get its quantum version. To this end one has to fix the gauge invariance of the action; we add the following gauge-fixing term, by using the Landau gauge condition:

$$S_{gf} = \int \mathrm{d}^2x\; \left(b^a\partial^\mu A^a_\mu - \partial^\mu\overline{c}^a(D_\mu c)^a\right) \tag{2.5}$$

where c, $\overline{c}^a$ and b^a are the ghost, the antighost and the Lagrange multiplier, respectively. The gauge fixed action

$$S = S_{YM} + S_{gf} = S_{top} + S_\phi + S_{gf} \tag{2.6}$$

Table 1.

	A^a_μ	ϕ^a	c^a	$\bar{c}^a$	b^a
Dim	0	1	0	0	1
Ng	0	0	1	−1	0

is invariant under the BRST transformations:

$$\begin{aligned} &sA^a_\mu = -\left(\partial_\mu c^a + g f^{abc} A^b_\mu c^c\right) \\ &sc^a = \tfrac{1}{2} g f^{abc} c^b c^c \\ &s\phi^a = g f^{abc} c^b \phi^c \\ &s\bar{c}^a = b^a \qquad sb^a = 0. \end{aligned} \tag{2.7}$$

The dimension and the ghost-number of the fields are displayed in table 1. Let us now focus on the following sector of the action (2.6):

$$S_{inv} = S_{top} + S_{gf} \tag{2.8}$$

corresponding to the quantized topological BF model. As already underlined, besides the BRST invariance (2.7), S_{inv} has a further symmetry; the so-called vector supersymmetry [9], which reads

$$\begin{aligned} &\delta_\mu A^a_\nu = 0 \\ &\delta_\mu \phi^a = -\varepsilon_{\mu\nu} \partial^\nu \bar{c}^a \\ &\delta_\mu c^a = -A^a_\mu \\ &\delta_\mu \bar{c}^a = 0 \\ &\delta_\mu b^a = \partial_\mu \bar{c}^a. \end{aligned} \tag{2.9}$$

In summary, we have

$$\begin{aligned} &sS_{inv} = 0 \\ &\delta_\mu S_{inv} = 0. \end{aligned} \tag{2.10}$$

In addition, the generators s and δ_μ give rise to the following Wess–Zumino supersymmetric algebra:

$$\begin{aligned} &\left\{s, \delta_\mu\right\} \phi^a = \partial_\mu \phi^a + \varepsilon_{\mu\nu} \frac{\delta S_{inv}}{\delta A^a_\nu} \\ &\left\{s, \delta_\mu\right\} A^a_\nu = \partial_\mu A^a_\nu - \varepsilon_{\mu\nu} \frac{\delta S_{inv}}{\delta \phi^a} \\ &\left\{s, \delta_\mu\right\} (c, b, \bar{c}) = \partial_\mu (c, b, \bar{c}) \end{aligned} \tag{2.11}$$

which, closing on-shell on the spacetime translations, allows for a supersymmetric interpretation of the two-dimensional BF model.

On the other hand, we can easily verify that, as expected, the metric-dependent term S_ϕ of the quantized 2DYM action in equation (2.6) breaks the vector supersymmetry invariance (2.9) of the topological sector. In fact,

$$\delta_\mu \mathcal{O} = -\varepsilon_{\mu\nu} \phi^a \partial^\nu \bar{c}^a \tag{2.12}$$

Table 2.

	s	δ_μ	τ	ξ_μ	$\eta_{\mu\nu}$	$\Omega_{\mu\nu}$
N_g	1	-1	0	1	2	1
Dim	1	0	0	0	0	-1

where we have defined $\mathcal{O} = \frac{1}{2}\phi^a\phi^a$. As previously remarked, our procedure in order to control the effects of this breaking is to introduce external fields as follows:

$$S_{\mathcal{O}} = \int \mathrm{d}^2x \left(\tau\mathcal{O} + \xi^\mu\delta_\mu\mathcal{O} + \tfrac{1}{2}\eta^{\mu\nu}\delta_\mu\delta_\nu\mathcal{O} + \tfrac{1}{2}\Omega^{\mu\nu}s\delta_\mu\delta_\nu\mathcal{O}\right) \tag{2.13}$$

with τ and ξ^μ being scalar and vector external fields, respectively, while $\eta^{\mu\nu}$ and $\Omega^{\mu\nu}$ are rank-2 antisymmetric external fields. Their canonical dimensions and ghost-numbers are displayed in table 2. It thus follows that the action

$$S_\tau = S + S_{\mathcal{O}} \tag{2.14}$$

is invariant under the modified nilpotent BRST transformations:

$$\begin{aligned}
&\tilde{s}A^a_\mu = -\left(D_\mu c\right)^a + \xi^\nu\varepsilon_{\nu\mu}\phi^a \\
&\tilde{s}c^a = \tfrac{1}{2}gf^{abc}c^bc^c \\
&\tilde{s}\phi^a = gf^{abc}c^b\phi^c \\
&\tilde{s}\bar{c}^a = b_a \qquad\qquad \tilde{s}b_a = 0 \\
&\tilde{s}\tau = -\partial_\mu\xi^\mu \qquad\qquad \tilde{s}\xi_\mu = 0 \\
&\tilde{s}\Omega_{\mu\nu} = -\eta_{\mu\nu} \qquad\qquad \tilde{s}\eta_{\mu\nu} = 0
\end{aligned} \tag{2.15}$$

$$\tilde{s}S_\tau = 0. \tag{2.16}$$

Moreover, remarkably in half, S_τ turns out to be left invariant by the following extended susy transformations:

$$\widetilde{\delta}_\mu c^a = -A^a_\mu \qquad\qquad \widetilde{\delta}_\mu A^a_\nu = 0 \tag{2.17}$$

$$\begin{aligned}
&\widetilde{\delta}_\mu\phi^a = -\varepsilon_{\mu\nu}\partial^\nu\bar{c}^a \qquad\qquad \widetilde{\delta}_\mu\bar{c}^a = 0 \\
&\widetilde{\delta}_\mu b^a = \partial_\mu\bar{c}^a \\
&\widetilde{\delta}_\mu\xi_\nu = -\delta_{\mu\nu}\left(1+\tau\right) \qquad\qquad \widetilde{\delta}_\mu\tau = 0 \\
&\widetilde{\delta}_\mu\eta_{\nu\kappa} = -\delta_{\mu\kappa}\xi_\nu + \delta_{\mu\nu}\xi_\kappa - \partial_\mu\Omega_{\nu\kappa} \\
&\widetilde{\delta}_\mu\Omega_{\nu\kappa} = 0
\end{aligned} \tag{2.18}$$

$$\widetilde{\delta}_\mu S_\tau = 0 \tag{2.19}$$

where $\delta_{\mu\nu}$ is the flat Euclidean metric. We therefore see that, as announced, we have been able to account for the breaking generated by the non-topological action S_ϕ by introducing suitable external fields. We are now ready to implement the $\tilde{s}$ and the $\widetilde{\delta}_\mu$-invariance of the action S_τ as Ward identities. This will be the task of the next section.

Table 3.

	$\gamma^{a\mu}$	L^a	ρ^a
Dim	1	1	0
Ng	−1	−2	−1

3. Ward identities

Following the standard BRST procedure [10], we add to expression (2.14) a new term S_{ext}, accounting for the nonlinear part of the modified BRST transformations (2.15):

$$S_{ext} = \int \mathrm{d}^2x \left(\gamma^{a\mu}\tilde{s}A^a_\mu + L^a\tilde{s}c^a + \rho^a\tilde{s}\phi^a\right) \tag{3.1}$$

where the external fields $\gamma^{a\mu}$, L^a and ρ^a have dimensions and ghost-numbers displayed in table 3. Therefore, the complete action

$$\Sigma = S_\tau + S_{ext} \tag{3.2}$$

obeys the Slavnov–Taylor identity

$$\mathcal{S}(\Sigma) = \int \mathrm{d}^2x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta\Sigma}{\delta\gamma^{a\mu}} + \frac{\delta\Sigma}{\delta\phi^a}\frac{\delta\Sigma}{\delta\rho^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + b_a\frac{\delta\Sigma}{\delta\bar{c}^a} - \partial_\mu\xi^\mu\frac{\delta\Sigma}{\delta\tau} - \frac{1}{2}\eta^{\mu\nu}\frac{\delta\Sigma}{\delta\Omega^{\mu\nu}}\right)$$
$$= 0. \tag{3.3}$$

Let us also introduce, for further use, the nilpotent linearized Slavnov–Taylor operator $\mathcal{B}_\Sigma$

$$\mathcal{B}_\Sigma = \int \mathrm{d}^2x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\gamma^{a\mu}} + \frac{\delta\Sigma}{\delta\gamma^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta\phi^a}\frac{\delta}{\delta\rho^a} + \frac{\delta\Sigma}{\delta\rho^a}\frac{\delta}{\delta\phi^a}\right.$$
$$\left. + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + b_a\frac{\delta}{\delta\bar{c}^a} - \partial_\mu\xi^\mu\frac{\delta}{\delta\tau} - \frac{1}{2}\eta^{\mu\nu}\frac{\delta}{\delta\Omega^{\mu\nu}}\right) \tag{3.4}$$

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0. \tag{3.5}$$

Turning now to the vector invariance $\widetilde{\delta}_\mu$, it is easily verified that, due to the introduction of the external fields $\gamma^{a\mu}$, L^a, ρ^a, it takes the form of a linearly broken Ward identity, namely

$$\mathcal{W}_\mu\Sigma = \Delta^{cl}_\mu \tag{3.6}$$

with

$$\mathcal{W}_\mu = \int \mathrm{d}^2x \left(\varepsilon_{\mu\nu}\rho^a\frac{\delta}{\delta A^a_\nu} - A^a_\mu\frac{\delta}{\delta c^a} - \varepsilon_{\mu\nu}\left(\gamma^{a\nu} + \partial^\nu\bar{c}^a\right)\frac{\delta}{\delta\phi^a} - L^a\frac{\delta}{\delta\gamma^{a\mu}}\right.$$
$$\left. + \partial_\mu\bar{c}^a\frac{\delta}{\delta b^a} - (1+\tau)\frac{\delta}{\delta\xi^\mu} + \frac{1}{2}\left(\delta^\alpha_\mu\xi^\beta - \delta^\beta_\mu\xi^\alpha - \partial_\mu\Omega\right)\frac{\delta}{\delta\eta^{\alpha\beta}}\right) \tag{3.7}$$

and

$$\Delta^{cl}_\mu = \int \mathrm{d}^2x \left(-\gamma^{a\nu}\partial_\mu A^a_\nu + L^a\partial_\mu c^a - \rho^a\partial_\mu\phi^a\right.$$
$$\left. -\varepsilon_{\mu\nu}\rho^a\partial^\nu b^a - L^a\xi^\nu\varepsilon_{\nu\mu}\phi^a + \gamma^a_\mu\xi_\nu\partial^\nu\bar{c}^a - \gamma^a_\nu\xi^\nu\partial_\mu\bar{c}^a + \gamma^a_\nu\gamma^a_\mu\xi^\nu\right). \tag{3.8}$$

We observe that the breaking term Δ^{cl}_μ, being linear in the quantum fields, is a purely classical breaking and will not get renormalized [10]. It is worth underlining that the final form of the

Ward identity (3.6) is the expected one, being indeed a common feature of the topological models, including in particular the associated classical breaking term [6–9, 11]. We also point out that, in the present case, the complete action Σ is constrained by a further linearly broken local Ward identity:

$$\mathcal{F}_{\mu\nu}(x)\Sigma = \Delta^{\mathcal{F}}_{\mu\nu}(x) \tag{3.9}$$

with

$$\begin{aligned}\mathcal{F}_{\mu\nu}(x) = \varepsilon_{\nu\mu}\phi^a(x)\frac{\delta}{\delta c^a(x)} + \varepsilon_{\nu\mu}L^a(x)\frac{\delta}{\delta\rho^a(x)} + \frac{\delta}{\delta\Omega^{\mu\nu}(x)} + \partial_\nu\frac{\delta}{\delta\xi^\mu(x)} - \partial_\mu\frac{\delta}{\delta\xi^\nu(x)}\\ + \left(\gamma^{a\nu} + \partial^\nu\bar{c}^a\right)\frac{\delta}{\delta A^{a\mu}(x)} - \left(\gamma^{a\mu} + \partial^\mu\bar{c}^a\right)\frac{\delta}{\delta A^{a\nu}}\end{aligned} \tag{3.10}$$

and

$$\Delta^{\mathcal{F}}_{\mu\nu}(x) = \gamma^a_\mu\partial_\nu b^a - \gamma^a_\nu\partial_\mu b^a. \tag{3.11}$$

In particular, the operators $\mathcal{B}_\Sigma$, $\mathcal{W}_\mu$ and $\mathcal{F}_{\mu\nu}(x)$ give rise to a closed algebra given by

$$\begin{aligned}\left\{\mathcal{B}_\Sigma, \mathcal{W}_\mu\right\} &= \mathcal{P}_\mu + \int \mathrm{d}^2x\, \xi^\nu\mathcal{F}_{\nu\mu}(x)\\ \left\{\mathcal{W}_\mu, \mathcal{W}_\nu\right\} &= \left\{\mathcal{W}_\mu, \mathcal{F}_{\sigma\rho}(x)\right\} = 0\\ \left\{\mathcal{B}_\Sigma, \mathcal{F}_{\sigma\rho}(x)\right\} &= \left\{\mathcal{F}_{\mu\nu}(x), \mathcal{F}_{\sigma\rho}(y)\right\} = 0\end{aligned} \tag{3.12}$$

where $\mathcal{P}_\mu$ is the functional operator of the spacetime translations, i.e.

$$\mathcal{P}_\mu = \sum_{(\text{all fields }\varphi)} \int \mathrm{d}^2x\, \partial_\mu\varphi\frac{\delta}{\delta\varphi}. \tag{3.13}$$

Finally, let us display the whole set of conditions which are usually imposed in the quantization of Yang–Mills theories in the Landau gauge [10], namely:

- The linearly broken ghost equation Ward identity

  $$\mathcal{G}^a\Sigma = \Delta^a \tag{3.14}$$

  with

  $$\mathcal{G}^a = \int \mathrm{d}^2x\left(\frac{\delta}{\delta c^a} + gf^{abc}\bar{c}^b\frac{\delta}{\delta b^c}\right) \tag{3.15}$$

  and

  $$\Delta^a = \int \mathrm{d}^2x\, g\, f^{abc}\left(\gamma^{b\mu}A^c_\mu - L^b c^c + \rho^b\phi^c\right). \tag{3.16}$$

- The gauge-fixing condition

  $$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^{a\mu} + \partial_\nu\Omega^{\mu\nu}\partial_\mu\bar{c}^a. \tag{3.17}$$

- The antighost equation

  $$\left(\frac{\delta}{\delta\bar{c}^a} + \partial_\mu\frac{\delta}{\delta\gamma^a_\mu}\right)\Sigma = \partial_\nu\eta^{\mu\nu}\partial_\mu\bar{c}^a + \partial_\nu\Omega^{\mu\nu}\partial_\mu b^a.$$

  As we shall see in the next section, the identities (3.3), (3.6) and (3.9) turn out to have far-reaching consequences, accounting for instance for the absence of non-trivial invariant counterterms.

4. Invariant counterterms

Following the set up of the algebraic renormalization [10] and making use of the general results on the cohomology of the Yang–Mills theories [12], it is not difficult to establish that the model and its Ward identities are renormalizable. Here, we shall limit ourselves only to state the final result, aiming to provide an algebraic understanding of the finiteness properties of 2DYM. Let us look then at the possible BRST-invariant counterterm Σ_c which may affect the ultraviolet behaviour of the model. We recall that Σ_c is an integrated local polynomial with dimension bounded by two. Making use of the Ward identities established in the previous section, Σ_c is found to obey the conditions

$$\frac{\delta \Sigma_c}{\delta b^c} = \mathcal{G}^a \Sigma_c = 0 \tag{4.1}$$

$$\left(\frac{\delta}{\delta \bar{c}^a} + \partial_\mu \frac{\delta}{\delta \gamma^a_\mu} \right) = 0 \tag{4.2}$$

$$\mathcal{B}_\Sigma \Sigma_c = 0 \tag{4.3}$$

$$\mathcal{F}_{\mu\nu}(x) \Sigma_c = 0 \tag{4.4}$$

$$\mathcal{W}_\mu \Sigma_c = 0. \tag{4.5}$$

From equation (4.1) it follows that Σ_c is independent of the Lagrange multiplier b^a and that it only depends on the differentiated ghost $\partial_\mu c^a$ [10]. Moreover, due to equation (4.2), the fields $\bar{c}^a$ and γ^a_μ enter through the combination [10]

$$\hat{\gamma}^a_\mu \equiv \gamma^a_\mu + \partial_\mu \bar{c}^a. \tag{4.6}$$

Finally, from conditions (4.3) and (4.4) it turns out that the most general BRST-invariant counterterm can be written as

$$\Sigma_c = \Xi + \mathcal{B}_\Sigma \tilde{\Xi} \tag{4.7}$$

where

$$\Xi = \eta \int \mathrm{d}^2 x \, \frac{\phi^2}{2} \tag{4.8}$$

with η being an arbitrary parameter and $\tilde{\Xi}$ denotes an integrated local polynomial with ghost number -1, representing the trivial part of the cohomology of the operator $\mathcal{B}_\Sigma$. Observe that, due to equation (4.4) and to the algebraic relations (3.12), we have

$$\mathcal{F}_{\mu\nu}(x) \mathcal{B}_\Sigma \tilde{\Xi} = \mathcal{B}_\Sigma \mathcal{F}_{\mu\nu}(x) \tilde{\Xi} = 0 \tag{4.9}$$

from which it follows that

$$\mathcal{F}_{\mu\nu}(x) \tilde{\Xi} = \mathcal{B}_\Sigma \Lambda_{\mu\nu}(x) \tag{4.10}$$

for some local polynomial $\Lambda_{\mu\nu}(x)$ of ghost number -3. We are left thus with a unique non-trivial BRST-invariant counterterm given by equation (4.8). Expression (4.8) is physically equivalent to the standard Yang–Mills counterterm $\int \mathrm{d}^2 x \, F^{a\mu\nu} F^a_{\mu\nu}$. This statement relies on the observation that the auxiliary field ϕ^a has, in fact, the role of $\varepsilon^{\mu\nu} F^a_{\mu\nu}$, as is implied by the equations of motion. Also, it should be observed that the coefficient η in equation (4.8) has the meaning of a possible ultraviolet renormalization of the gauge coupling constant g compatible with the BRST invariance.

It remains now to impose the final constraint (4.5). Making use of the algebraic relations (3.12), it follows that

$$
\begin{aligned}
\mathcal{W}_\mu \mathcal{B}_\Sigma \tilde{\Xi} &= -\mathcal{B}_\Sigma \mathcal{W}_\mu \tilde{\Xi} + \left\{\mathcal{B}_\Sigma, \mathcal{W}_\mu\right\} \tilde{\Xi} \\
&= -\mathcal{B}_\Sigma \mathcal{W}_\mu \tilde{\Xi} + \int \mathrm{d}^2x\, \xi^\nu \mathcal{F}_{\nu\mu} \tilde{\Xi} \\
&= -\mathcal{B}_\Sigma \left(\mathcal{W}_\mu \tilde{\Xi} + \int \mathrm{d}^2x\, \xi^\nu \Lambda_{\nu\mu} \right)
\end{aligned}
\tag{4.11}
$$

where use has been made of equation (4.10) and of the fact that $\mathcal{B}_\Sigma \xi_\mu = 0$. Therefore, from the requirement of invariance under the Ward operator $\mathcal{W}_\mu$, we have

$$
\mathcal{W}_\mu \Sigma_c = \mathcal{W}_\mu \Xi - \mathcal{B}_\Sigma \left(\mathcal{W}_\mu \tilde{\Xi} + \int \mathrm{d}^2x\, \xi^\nu \Lambda_{\nu\mu} \right) = 0 \tag{4.12}
$$

i.e.

$$
-\eta \int \mathrm{d}^2x\, \varepsilon_{\mu\nu} \hat{\gamma}^{a\nu} \phi^a = \mathcal{B}_\Sigma \left(\mathcal{W}_\mu \tilde{\Xi} + \int \mathrm{d}^2x\, \xi^\nu \Lambda_{\nu\mu} \right) \tag{4.13}
$$

implying that $\int \mathrm{d}^2x\, \varepsilon_{\mu\nu} \hat{\gamma}^{a\nu} \phi^a$ is a trivial element of the integrated cohomology of $\mathcal{B}_\Sigma$. However, it can be shown that this term cannot actually be cast in the form of a pure $\mathcal{B}_\Sigma$-variation, as it identifies a non-trivial element of the integrated cohomology of $\mathcal{B}_\Sigma$, namely

$$
\int \mathrm{d}^2x\, \varepsilon_{\mu\nu} \hat{\gamma}^{a\nu} \phi^a \neq \mathcal{B}_\Sigma\text{-variation.} \tag{4.14}
$$

The only way out is thus

$$
\eta = 0 \tag{4.15}
$$

meaning that there is no non-trivial BRST-invariant counterterm compatible with the vector Ward identity (3.6), providing therefore a simple algebraic understanding of the well known ultraviolet finiteness properties of 2DYM[4]. It is useful remarking that the algebraic proof of the absence of non-trivial counterterms given here follows exactly the same lines of the proofs of the ultraviolet finiteness of the topological models [6–9, 11], emphasizing, in particular, the pivotal role of the vector Ward identity (3.6).

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[4] We shall not be concerned here with possible infrared pathologies of the model, our aim being that of analysing the ultraviolet region. A useful BRST-invariant infrared regularization could be introduced along the lines developed in [9, 13] and already successfully applied in the case of the 2D topological BF model [9].

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